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Tunnelling in crossed electric and magnetic fields

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Abstract : A new model of tunnelling in crossed electric and magnetic fields proposed by Roy and Ghosh [1] has been used to establish an analytical expression for tunnel current density. Current-Voltage characteristics in M-I-M junctions and tunnel diodes have been computed which are in good qualitative agreement with observations. A method of computing tunnelling time has also been suggested.

Keywords : Tunnel current density, tunnelling time, crossed electric and magnetic fields

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1. Introduction

The study of this problem started after Calawa *et al* [2] reported their observations according to which the tunnel current was found to decrease with increasing magnetic fields in InSb tunnel diodes placed in crossed electric and magnetic fields. Roth *et al* [3] studied the influence of such fields on Ge tunnel diodes and their observations were identical with those of the former. Several attempts were made later on to understand the mechanism of tunnelling in crossed electric and magnetic fields, important among them being those by Roth *et al* [3], Zawadzki and Lax [4] and Aronov and Pikus [5].

Roth and his co-workers [3] proposed a phenomenological theory assuming a quadratic dependence of Kane voltage on the magnetic field. According to them, the shift in the Kane voltage V_k due to the magnetic field is defined as

$$\Delta V_k = \alpha \mathcal{H}^2 \tag{1}$$

where α is a constant and $\mathcal{H}$ is the applied magnetic field. They obtained an expression for the tunnel current given by

$$Jd\,(\mathcal{H}) = \left(\frac{AB}{0.30}\right) \exp\,(BV)\,[V - V_k - \Delta V_k\,(\mathcal{H})]^2 \tag{2}$$

where A and B are constants and V is the applied bias. They also obtained the change in the tunnel current due to the transverse magnetic field given by

$$\begin{aligned} \Delta J_d\,(\mathcal{H}) &= K\mathcal{H}^2\,[2 - \alpha\mathcal{H}^2/(V - V_k)] \text{ for } (V - V_k - \alpha\mathcal{H}^2) > 0 \\ &= 0 \qquad\qquad\qquad\qquad\quad \text{for } (V - V_k - \alpha\mathcal{H}^2) < 0 \end{aligned} \tag{3}$$

where K is an arbitrary parameter assumed to be dependent on the voltage.Plotting $\Delta J_d(\mathcal{H})$ against the magnetic field for several bias voltages, they have shown that their theoretical curves deviate from the quadratic behaviour for higher magnetic fields in case of lower biases whereas they claim good agreement for higher biases.

Zawadzki and Lax [6] showed that tunnelling is not possible in the one-band effective mass approximation. A two-band model has been suggested by them.Taking the energy-momentum relation for two interacting bands suggested by Kane [7],

$$E(k) = \pm \left[\left(\frac{E_g}{2} \right)^2 + E_g \frac{\hbar^2 k^2}{2m} \right]^{1/2} \tag{4}$$

and using the principles of classical relativistic mechanics, it has been shown that for $\mathcal{E} < \mathcal{H}$ ($\mathcal{E}$ is the electric field) the motion is magnetic field type and for $\mathcal{E} > \mathcal{H}$, it is electric field type. They have written the Hamiltonian for an electron in a periodic potential in the presence of a crossed field as

$$H = \frac{1}{2m_0} \left(p + \frac{e}{c} A \right)^2 + e\mathcal{E} \cdot r + V(r) \tag{5}$$

where e is the electronic charge and c is the velocity of light. Solving the Schroedinger's equation after using W.K.B. approximation they have obtained the band picture which shows that the application of the magnetic field curves the allowed regions away from each other which effectively increases the band gap which in turn decreases the tunnel current His expression for the tunnel current is given by

$$J(\mathcal{H}) = \frac{e^2 \mathcal{E}^2}{3\pi^2 \hbar^2} \left(\frac{2m}{E_g} \right)^{1/2} \exp \left[-\frac{\pi}{2\hbar e \mathcal{E}_{\text{eff}}} \left(\frac{m}{2} \right)^{1/2} E_g^{3/2} \right] \tag{6}$$

where $$\mathcal{E}_{\text{eff}}^2 = \mathcal{E}^2 - \frac{\mathcal{H}^2 E_g}{2mc^2} \tag{7}$$

Thus we find that increasing the magnetic field decreases $\mathcal{E}_{\text{eff}}$ which has the effect of decreasing the tunnel current $J(\mathcal{H})$.

Aronov and Pikus [5] have used a two-band equation which is similar in form to the Dirac equation and treating the problem relativistically, they have obtained an expression for the tunnel current given by

$$J(\mathcal{H}) = \frac{e^2 (\mathcal{E}'^2 - \mathcal{H}^2)}{36\pi\hbar^2 s} \exp \left[- \frac{\pi m^2 s^3}{e\hbar (\mathcal{E}'^2 - \mathcal{H}^2)^{1/2}} \right] \tag{8}$$

where $s = \left(\frac{E_g}{2m} \right)^{1/2}$ and $\mathcal{E}'$ and $\mathcal{H}$ are effective electric and magnetic fields. Eq. (8) is found to be similar in nature to eq. (6).

In the present paper, a completely different approach has been adopted to compute the tunnel current in crossed electric and magnetic fields. Roy and his coworkers [8, 9] have pointed out the inadequacies of the conventional theory of tunnelling and have proposed a completely new approach to the entire problem of tunnelling. The conventional models treat the electron as a particle inside the potential barrier which is untenable because one cannot know anything about the particle so long as it is negotiating the barrier. It is only its probability density which can be said to be continuous through the barrier and the new model is based upon this approach. In the next section, an expression for the tunnel current-density under the influence of crossed electric and magnetic fields has been derived based on the ideas of the new model.

2. Tunnel current density in crossed electric and magnetic fields based on the new approach

Roy and Ghosh [1] have shown that when a tunnel device is placed in crossed electric and magnetic fields with the electric field F_x along the x direction and the magnetic field B_z along the Z-direction, the barrier wave function for the electron may be written as

$$\psi(x, t) = a_l(t)\,\alpha \exp\left[\left\{-\sqrt{\frac{e}{\hbar}(B_z + \lambda)}\right\}x\right] \exp(-i\omega_l t)$$

$$+ a_r(t)\,\beta \exp\left[\left\{+\sqrt{\frac{e}{\hbar}(B_z + \lambda)}\right\}x\right] \exp(-i\omega_r t) \tag{9}$$

where $\lambda = \dfrac{2m^*}{e\hbar}(V_0 - E)$, V_0 being the maximum height of the barrier; m^* is the effective mass of the electron in the barrier, $a_l(t)$ and $a_r(t)$ are time-dependent coefficients and α and β are coefficients obtained in the time-independent treatment of the barrier problem.

$$\omega_{l,r} = \frac{E_{l,r}}{\hbar}, \quad \text{where } E \text{ is the energy of the electron.}$$

Writing ω_l and ω_r in the two terms on the right hand side of eq. (9) is one of the main contentions of the new model [8, 9]. The electron, which possesses a negative kinetic energy inside the barrier cannot be treated as a particle. Neither has it the character of a wave because the two solutions inside the barrier are exponentially growing and decaying functions. So the electron is treated as something intermediate between a particle and a wave. It interacts with the barrier potential and there is every chance that its incident energy E_l undergoes change to energy E_r during the process of tunnelling.

Taking $a_l(t) = 1$ and $a_r(t) << 1$, we may write

$$\psi(x, t) = \alpha \exp\left[\left\{-\sqrt{\frac{e}{\hbar}(B_z + \lambda)}\right\}x\right] \exp(-i\omega_l t)$$

$$+ a_r(t)\,\beta \exp\left[\left\{+\sqrt{\frac{e}{\hbar}(B_z + \lambda)}\right\}x\right] \exp(-i\omega_r t) \tag{10}$$

The time-dependent position probability density may be written as

$$|\psi|^2 = |\alpha|^2 \exp\left[\left\{-2\sqrt{\frac{e}{\hbar}(B_z + \lambda)}\right\}x\right] + a^*_r(t)\,\beta^*\alpha \exp(-i\omega_{rl} t)$$

$$+ a_r(t)\,\beta\alpha^* \exp(-i\omega_{lr} t) + |a_r(t)|^2\,|\beta|^2 \exp\left[\left\{2\sqrt{\frac{e}{\hbar}(B_z + \lambda)}\right\}x\right] \tag{11}$$

where $\omega_{lr} = \frac{E_l - E_r}{\hbar}$.

Using the equation of continuity, the tunnel current density may be written as

$$J_t = e\int_{x_1}^{x_2}\left\{\frac{\partial}{\partial t}|\psi|^2\right\}_{t=\tau} dx \tag{12}$$

where τ is the tunnelling time and x_1 and x_2 are the barrier extremeties. Evaluating the integral of eq. (12) one finally obtains

$$J_t = J_{01}\frac{\sin\omega_{lr}\tau}{\omega_{lr}\tau} + J_{02}\sin(\omega_{lr}\tau + \theta) \tag{13}$$

where

$$J_{01} = \frac{4e\hbar^2\tau|\alpha|^2\chi_B^3}{m^{*2}}\exp(-2\chi_B W)$$

and

$$J_{02} = \frac{4e\hbar w|\alpha|^2\chi_B^2}{m^*}\exp(-2\chi_B W) \tag{14}$$

$W = x_2 - x_1$ is the width of the barrier and θ is a phase factor. Also

$$\chi_B = \left[\frac{eBz}{\hbar} + \frac{2m^*}{\hbar^2}(V_o - E)\right]^{1/2} \tag{15}$$

Eq. (13) gives us a spectrum of tunnel current density. This expression suggests that the emerging particle will not necessarily have the same energy at which it was incident. There is a definite probability for a particle incident with energy E_1 emerging with a different energy E_r. The problem of computing the tunnel current density becomes a bit complicated because of distribution of electronic states on both the sides of the barrier. Also the incident electrons are incoherent. To tide over these difficulties, eq. (13) is summed over all possible values of E_r ranging from $-\infty$ to $+\infty$. The infinite range of E_r has to be considered to take into account the complete randomness in the incidence of electrons of particular energy. Due to the conjugate relationship between E and t, the randomness or uncertainty in time is transferred to the uncertainty in energy i.e. if we regard that all the electrons are incident simultaneously ($\Delta t = 0$), then the energy uncertainty becomes infinite and hence an infinite range for E_r. Therefore, for electrons incident with a particular energy E_1 we may write [8, 9]

$$J_t(E_l) = \sum_r J_t = J_{01}\sum_{E_r=-\infty}^{+\infty}\frac{\sin(\omega_{lr}\tau)}{(\omega_{lr}\tau)} + J_{02}\sum_{E_r=-\infty}^{+\infty}\sin(\omega_{lr}\tau + \theta) \tag{16}$$

To convert the above summations into integrals, we write

$$E_l - E_r = n\varepsilon \tag{17}$$

where ε is the separation of consecutive energy levels E_r and n is an integer having both positive and negative values. Since the levels are very closely spaced, one may conveniently write the above summations in the form of integreals as given below :

$$J_t(E_l) = J_{01} \int_{-\infty}^{+\infty} \frac{\sin\left(\frac{n\varepsilon\tau}{\hbar}\right)}{\left(\frac{n\varepsilon\tau}{\hbar}\right)} dn + J_{02} \int_{-\infty}^{+\infty} \sin\left(\frac{n\varepsilon\tau}{\hbar} + \theta\right) dn \tag{18}$$

which yields

$$J_t(E_l) = \frac{\pi\hbar}{\varepsilon\tau} J_{01} \tag{19}$$

The differential tunnel current density at low temperatures may be written as

$$dJ_{t\,(B)} = J_t(E)\,\rho_l(E)\,dE \tag{20}$$

where $\rho_l(E)$ is the density of states on the incident side, E_l having been replaced by E.

Writing $\frac{1}{\varepsilon} = \rho_r(E)\,\Omega$

where Ω is the volume of the electrode to which tunnelling takes place, we may write [8]

$$dJ_{t\,(B)} = \frac{\pi\hbar\Omega}{\tau} J_{01}\,\rho_l(E)\,\rho_r(E)\,dE \tag{21}$$

$$\text{where } \rho_l(E) = \rho_r(E) = 4\pi\left(\frac{2m^*}{\hbar^2}\right)^{3/2} E^{1/2} \tag{22}$$

Hence the total tunnel current density may be written as

$$J_t(B) = \int dJ_t(B) \tag{23}$$

After substituting for J_{01}, $\rho_l(E)$ and $\rho_r(E)$ one finally obtains

$$J_{t\,(B)} = \frac{2^{11}\pi^3 m^* e\Omega}{V_0\hbar^3} \int \chi_B{}^3 \exp(-2\chi_B W) E^2\,dE \tag{24}$$

For large barriers we may neglect E in comparison with V_0 so that χ_B and $\exp(-2\chi_B W)$ become independent of E in the integrand of eq. (24). Hence

$$J_t(B) = \frac{2^{11}\pi^3 m^* e\Omega\chi_B{}^3}{V_0\hbar^3} \exp(-2\chi_B W) \int E^2\,dE \tag{25}$$

$$\text{where now } \chi_B = \left(\frac{2m^*}{\hbar^2} V_0 + \frac{eB_z}{\hbar}\right)^{1/2} \tag{26}$$

It is the maximum height of the barrier which is influencing the tunnelling process and not its shape. This is because in the new approach, the barrier is regarded to introduce an uncertainty in the energy of the incident electron equal to its maximum height. This simplifies the problem of computation of the tunnel current density. Although the shape of the barrier in the case of an M-I-M junction is a rectangular one and that in the case of a tunnel diode is a triangular one, the maximum height of the barrier in the former case is taken as V_0 and in the latter case as E_g, the band gap energy.

For the sake of integration appearing in eq. (25), the zero of the energy scale is taken at the Fermi level of the electrode to which the tunnelling is taking place. Thus taking the limits to be 0 and eV in the case of M-I-M junctions and 0 and ΔE in the case of tunnel diodes, we have [10]

$$\left.\begin{aligned} [J_t(B)]_{\mathrm{MIM}} &= A\,(eV)^3 \\ \text{and} \qquad [J_t(B)]_{\text{tunnel diode}} &= A\,(\Delta E)^3 \end{aligned}\right\} \tag{27}$$

where,

$$A = \frac{2^{11}\pi^3 m^* e\Omega\chi_B{}^3}{3V_0 h^3} \exp\left(-2\chi_B W\right) \tag{28}$$

and $\Delta E = E_1 + E_2 - eV$, E_1 and E_2 being Fermi level degeneracies on the n and p sides of the tunnel diode respectively.

Eq. (24) may be applied to find out the tunnel current density for a barrier of any height but in that case χ_B cannot be independent of E and will have to be included in the integral. This yields a lengthy expression but it is qualitatively similar to eq. (27).

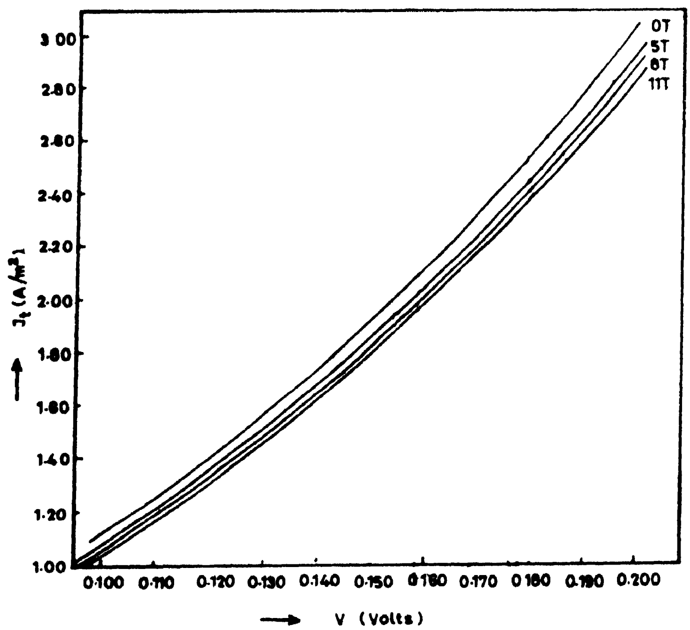

Figure 1. $J_t(B) - V$ (reverse) curve for a Ge tunnel diode in crossed electric and magnetic fields (large barriers)

The total tunnel current may be computed by taking the product of $J_t(B)$ with the area of the electrode to which the tunnelling is taking place.

Considerations of the perpendicular component of energy may also be incorporated by splitting ρ (E) and E into one-dimensional and two-dimensional components. This too does not change the results significantly.

3. Conclusions

$J_t(B) - V$ characteristics for a reverse biased Ge tunnel diode have been plotted in Figure 1 for different magnetic fields. As can be seen from the figure the current rises monotonically with bias but decreases with increasing magnetic fields. Figure 2 shows the $J_t(B) - V$ characteristics for a forward biased InSb tunnel diode for different values of the magnetic fields. It is seen that the current decreases with increasing magnetic fields but the peak voltage does not shift. Both the results pointed out above agree qualitatively with the observations of Roth *et al* [3] and Calawa *et al* [2]. The characteristics for an M-I-M junction show that there is a small change even for high magnetic fields which may be possibly the reason for the early workers not having reported on such systems.

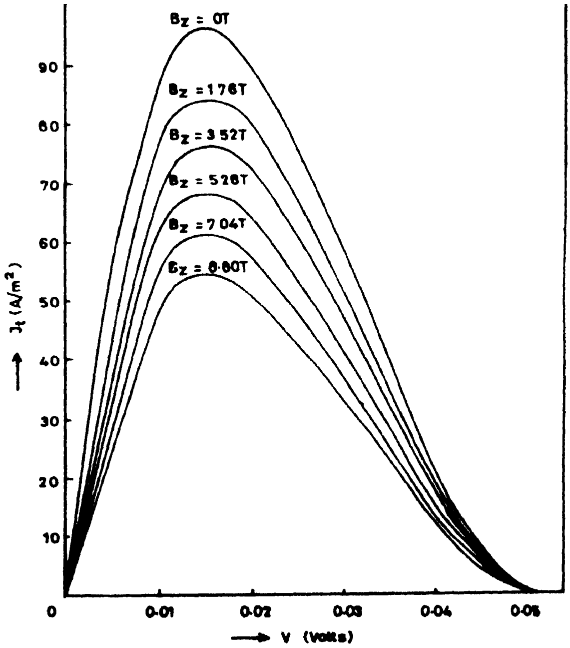

Figure 2. $J_t(B) - V$ (forward) curve for an InSb tunnel diode in crossed electric and magnetic fields (large barriers).

The identity of the magnetic field dependent exponential obtained in this work and that in Zawadzki's work may be shown in the following manner

$$\exp(-2\chi_B W) = \exp\left[-2\left\{\frac{2m^*}{\hbar^2} V_0 + \frac{eB_z}{\hbar}\right\}^{1/2} W\right]$$

$$= \exp\left[\frac{-4\sqrt{2}m^{*1/2} V_0^{3/2}}{2e\hbar\varepsilon}\left\{1 + \frac{eB_z\hbar}{2m^* V_0}\right\}^{1/2}\right]$$

$$= \exp\left[-\frac{4\sqrt{2m^*}^{1/2} V_o^{3/2}}{2e\hbar \varepsilon_{\mathrm{eff}}}\right] \tag{29}$$

where $W = \dfrac{V_o}{e\mathcal{E}}$ has been substituted.

The effective electric field appearing in eq. (29) may be expressed as

$$\mathcal{E}_{\mathrm{eff}} = \mathcal{E}\left[1 + \frac{eB_z\hbar}{2m^*V_o}\right]^{1/2}$$

and $$\mathcal{E}^2_{\mathrm{eff}} = \mathcal{E}^2\left[1 - \frac{eB_z\hbar}{2m^*V_o}\right]^{-1} \tag{30}$$

after binomial expansion (since $eB_z\hbar << 2m^*V_o$).

Substituting $V_o = E_g$ for a tunnel diode, we have

$$\mathcal{E}^2_{\mathrm{eff}} = \mathcal{E}^2\left(1 - \frac{eB_z\hbar}{2m^*E_g}\right) \tag{31}$$

Hence, $$\exp(-2\chi_B W) = \exp\left(-\frac{4\sqrt{2m^*}^{1/2} E_g^{3/2}}{2e\hbar\mathcal{E}_{\mathrm{eff}}}\right) \tag{32}$$

We find that $\mathcal{E}_{\mathrm{eff}}$ in our case is different from the one appearing in eq. (7) obtained by Zawadzki and Lax [6]. The magnetic field dependent term is found to be proportional to B_z in our case whereas it is proportional to $\mathcal{H}^2$ in Zawadzki's treatment.

From eq. (31) we find the value of cut-off magnetic field

$$(B_z)_{\text{cut-off}} = \frac{2m^*E_g}{e\hbar} \tag{33}$$

at which $\mathcal{E}_{\mathrm{eff}} = 0$ which makes the tunnel current vanish.

Defining the critical cyclotron frequency ω_c (at which the tunnel current ceases) as

$$\omega_c = \frac{e\,(B_z)_{\text{cut-off}}}{m^*} \tag{34}$$

it can be shown from eq. (33) that

$$\omega_c = \frac{2E_g}{\hbar} = \frac{2}{\hbar/E_g} = \frac{2}{\tau} \tag{35}$$

because tunnelling time τ is defined as

$$\tau = \frac{\hbar}{\text{barrier height}} = \frac{\hbar}{E_g} \text{ in the new approach.}$$

This conclusion is exactly the same as has been shown by Roy and Ghosh [1]. Thus a theoretical method for computing the tunnelling time is obtained.

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